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Technical Memorandum

NULL-STEERING PERFORMANCE DEGRADATION DUE TO SENSOR ELEMENT POSITION MEASUREMENT ERRORS

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ABSTRACT

This paper examines the performance degradation of a linearly-constrained, optimum beamformer designed to steer nulls in the direction of interfering sources. The proposed beamformer is optimum in the sense that beamformer response to ambient noise uncorrelated with the signals and interferers is minimized. Beampattern nulls and the maximum response axis (MRA) are steered in any desired direction by applying the method of Lagrange multipliers to constrain the beamformer directional response and then the output power is minimized subject to these constraints. The receiving array is allowed arbitrary spatial configurations in the analytical treatment, but specific, near-planar configurations are used in computer simulations of the algorithm. The measure of performance used is the ratio of output power due to sources in the directions of steered nulls to the power due to a source in the direction of the steered MRA. This ratio is examined as the error in measurement of sensor locations increases. An upper bound on this ratio is derived in terms of the maximum sensor element position measurement error. Computer simulations indicate that the upper bound derived is conservative.

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INTRODUCTION

This paper describes a study of the effect of errors in the estimated (or assumed) sensor positions of a receiving array on the ability of a beamformer to place the maximum response axis (MRA) and a number of nulls of a beam pattern in specified directions while minimizing beampattern sensitivity to distortion of the array. The spatial arrangement of the array of sensor elements is arbitrary in the analytical treatment, but specific, realistic, volumetric spatial configurations are used in computer simulations of beampatterns and in evaluations of the degradation in null steering capability.

The null-steering algorithm is based primarily on work by Frost[1] in which is described an algorithm for implementing linearly-constrained, adaptive beamforming. That is, the array output, as a function of the direction of incidence for planewave signals, is subjected to constraints specifying particular amplitudes at certain directions, and then the set of weights (i.e., the array shading) is calculated so as to both satisfy the beam pattern constraints and minimize the susceptibility to element position errors.

The measure of degradation used is similar to the output interference-to-signal ratio (OISR) used by Friedlander and Porat[2] in a first-order performance analysis of a *two-step, null-steering* algorithm. That paper presents an asymptotic expression which illustrates the dependence of the mean OISR on the angular separation of the desired signal source and an interfering source. The expectation is taken over the direction-of-arrival vector parameters, which are assumed to be suitably behaved random variables.

In the present paper the magnitude of the array output in the directions of the steered nulls is compared with the array output in the steered direction of the MRA as the magnitude of the sensor displacements is increased. The analysis shows that the power output from the sources in the direction of steered nulls increases while the power contribution from the source in the steered direction of the MRA decreases. Thus one may consider the total contribution to the array output power from the directions of the steered nulls as interference power that is zero when the null-steering algorithm is effective and which increases as the effectiveness of null steering is degraded. Examples of interference include enemy jamming signals (in the case of fighter aircraft, for instance), known, local merchant shipping or escort ships in a battle group, and own-ship radiated noise in the case of a bi-static or stand-off towed array sonar system. The array output in the direction of the steered MRA may be considered as the desired signal output. The ratio of these expected power outputs may be considered a measure of

the decreasing effectiveness of null steering as sensor element position errors increase. This ratio is presented as a function of the sensor element position error standard deviation in wavelengths.

Following sections of this paper describe the assumptions made in modeling the signal and noise fields, some terminology used to describe beamformer output, the principles of linearly constrained optimization as applied to array signal processing, and the performance degradation due to sensor element position measurement errors. Next the computer implementation of this study and analysis of the simulation results are presented as relate to the acoustic performance of a towed array sonar.

SIGNAL MODEL

Let $r_i(t)$ be the complex representation of the output of the ith sensor of an array of N sensors due to M narrow-band sources and ambient noise. The sensors may have arbitrary directional response and locations. Thus we may write

$$r_i(t)=\sum_{j=1}^{M} s_j(t)exp[\mathbf{j}\omega(t-t_{ij})]+n_i(t), \text{ for } i=1,...,N \tag{1}$$

where the $s_j(t)$ are complex envelopes of the M signals, boldface j is the imaginary unit, ω is radian frequency, t_{ij} is the jth signal propagation delay between the location of the ith sensor and some convenient reference, and $n_i(t)$ is the ambient noise.

The $s_j(t)$ are assumed to be slowly-varying, zero-mean, weakly-stationary random processes with variance p_j. The M sources are assumed to be not correlated with each other or with the ambient noise. Accordingly, the first and second order statistics for the $s_j(t)$ are given by

$$< s_j(t) > =0, \text{ for } j=1,\ldots,M \tag{2}$$

and

$$<s_j(t)s_k(t)>=\delta_{jk}p_j, \text{ for } j,k=1,\ldots,M \tag{3}$$

where angular brackets denote the expected value operator, and δ_{jk} is the Kronecker delta.

OUTPUT OF THE UNDISTORTED ARRAY

The output of all N sensors may be written concisely using vector notation as

$$\boldsymbol{r}(t)=\sum_{j=1}^{M} s_j(t)\boldsymbol{a}_j exp(j\omega t)+\boldsymbol{n}(t) \tag{4}$$

where

$$\boldsymbol{r}=[r_1(t),\ldots,r_N(t)]^{\mathrm{T}},$$

$$\boldsymbol{a}_j=[exp(-j\omega t_{1j}),\ldots,exp(-j\omega t_{Nj})]^{\mathrm{T}},\text{ for } j=1,\ldots,M,$$

and

$$\boldsymbol{n}(t)=[n_1(t),\ldots,n_N(t)]^{\mathrm{T}}.$$

The superscript $^{\mathrm{T}}$ denotes transposition. The vectors $\boldsymbol{a}_j$ are called direction-of-arrival (DOA) vectors in the technical literature because the components give complex representations of the signal phase at each sensor and thus provide information which can be used to determine the propagation direction.

The beamformer output signal may be written as an inner product representing the weighted sum of the N sensor outputs

$$b(t)=\boldsymbol{w}^H\boldsymbol{r}(t)=\sum_{j=1}^{M} s_j(t)\boldsymbol{w}^H\boldsymbol{a}_j exp(j\omega t)+\boldsymbol{w}^H\boldsymbol{n}(t) \tag{5}$$

where the complex vector of weights $\boldsymbol{w}$ defines the beamformer, and the superscript ^{H} denotes simultaneous transposition and complex conjugation. The beamformer power output is found by taking the expectation of the squared modulus of (5), given by

$$< b(t)b^*(t) > = < \boldsymbol{w}^H\boldsymbol{r}(t)\boldsymbol{r}(t)^H\boldsymbol{w}> = \boldsymbol{w}^H<\boldsymbol{r}(t)\boldsymbol{r}(t)^H>\boldsymbol{w} \tag{6}$$

where superscript * denotes complex conjugation.

Defining the noise cross correlation matrix as $\mathrm{Q}=<\boldsymbol{n}(t)\boldsymbol{n}^H(t)>$ and using (4) and (3), the right-hand side of (6) may be expanded to give

$$< b(t)b^{*}(t) > = \sum_{j=1}^{M} p_j \left\| w^H a_j \right\|^2 + w^H Q w \tag{7}$$

The summation in (7) represents the beamformer power output due to the M signals, the second term represents the power due to ambient noise only, and the vertical bars denote absolute value or magnitude of scalar arguments.

ARRAY ELEMENT POSITION ERRORS

Let the sensor-element assumed positions be given by or measured as the 3x1 Cartesian coordinate vectors x_i, for i=1,..., N. Let the actual array element positions be denoted by

$$x_i' = x_i + \alpha_i, \text{ for } i=1,\ldots,N \tag{8}$$

where α_i are 3x1 vectors of position error. Due to the position errors included in (8), the signal propagation delays used in the actual DOA vectors differ from the assumed values. For the case of far-field sources, the assumed propagation delays are

$$t_{ij} = u_j^{\mathrm{T}} x_i / c \tag{9}$$

where u_j is the unit vector pointing in the direction of propagation from the jth far-field source, and c is the propagation speed. Using (8), the actual propagation delay is given by

$$t_{ij}' = u_j^{\mathrm{T}} x_i' / c = (u_j^{\mathrm{T}} x_i + u_j^{\mathrm{T}} \alpha_i)/c \tag{10}$$

Multiplying (9) and (10) by the radian frequency ω gives, respectively, the assumed and actual phase relative to the phase at the reference as

$$\theta_{ij} = \omega t_{ij} \tag{11}$$

and

$$\theta_{ij}' = \omega t_{ij}' = \theta_{ij} + \phi_{ij} \tag{12}$$

where the $\phi_{ij}=\omega \boldsymbol{u}_j^{\mathrm{T}}\boldsymbol{\alpha}_i/c$ represent the phase errors due to element position errors.

This changes the entries of the DOA vectors to the form

$$\exp(-j\theta_{ij}')=exp[-j(\theta_{ij}+\phi_{ij})] \tag{13}$$

Using (13) we may write a matrix expression for the actual DOA vectors $\boldsymbol{a}_j'$ in terms of the assumed DOA vectors:

$$\boldsymbol{a}_j' = B_j\boldsymbol{a}_j \tag{14}$$

where B_j=diag[$exp(-j\phi_{1j}),...,exp(-j\phi_{Nj})$] is an NxN diagonal matrix with entries representing the complex exponential of signal phase errors, for the jth signal, due to displacement of the sensor array elements. We shall next examine the effect of these position errors on the beamformer output.

OUTPUT OF THE DISTORTED ARRAY

If we treat the array element position errors as random variables, the beamformer power output assumes a different form than given by (7) due to the appearance of the matrices B_j in equation (5). Substituting $\boldsymbol{a}_j'$ for $\boldsymbol{a}_j$ in (5) gives the actual beamformer output signal as

$$b(t)'=\boldsymbol{w}^H\boldsymbol{r}(t)'=\boldsymbol{w}^H\sum_{j=1}^{M}s_j(t)B_j\boldsymbol{a}_j exp(j\omega t)+\boldsymbol{w}^H\boldsymbol{n}(t)' \tag{15}$$

where primed symbols denote the quantities as previously defined excepting inclusion of the effects due to unknown displacement of the sensor elements from the assumed positions.

The power output of the distorted array is the expected value of the squared modulus of (15) or

$$< b(t)'b^*(t)' > =\boldsymbol{w}^H<\sum_{j=1}^{M}s_j(t)B_j\boldsymbol{a}_j\sum_{k=1}^{M}s_k(t)\boldsymbol{a}_k^H B_k^H>\boldsymbol{w} + \boldsymbol{w}^H\mathrm{Q}'\boldsymbol{w} \tag{16}$$

Although the ambient noise cross-correlation matrix Q' in fact differs from Q, our attention is focussed on the first term in order to assess the effects of array perturbations on the beamformer signal output.

The product of sums inside the expectation brackets of (16) result in terms such as

$$< s_j(t)s_k(t)B_j\mathbf{a}_j\mathbf{a}_k{}^H B_k{}^H > = \delta_{jk}p_j < B_j\mathbf{a}_j\mathbf{a}_k{}^H B_k{}^H > \tag{17}$$

where the assumed statistical properties of the M signals, given in (2) and (3), have been used in the right-hand side. Evaluation of the remaining expectation on the right-hand side of (17) is based on work by Gilbert and Morgan[3] and proceeds as follows. For notational convenience the subscript j denoting a particular DOA vector $\mathbf{a}_j$ (and corresponding diagonal matrix B_j) is dropped. Thus boldface $\mathbf{a}$ may represent any possible DOA vector, of which the M vectors $\mathbf{a}_j$ are merely a finite subset, and B represents the corresponding diagonal matrix, so that we may write

$$B\mathbf{a}\mathbf{a}^H B^H = B(\mathbf{a}\mathbf{a}^H\text{-}I)B^H + BB^H = B(\mathbf{a}\mathbf{a}^H\text{-}I)B^H + I \tag{18}$$

where I is the identity matrix. The first term on the right-hand side of (18) is an NxN matrix with zeros on the main diagonal. The off-diagonal entries have form $exp[j(\theta_k - \theta_i)]exp[j(\phi_k - \phi_i))]$ where again, the subscript j denoting a particular signal has been omitted so that θ_i and ϕ_i signify, respectively, the assumed signal phase and the phase error at the ith sensor for any possible signal propagation direction, and the subscripts i and k denote the row and column indices, respectively, of the matrix entries.

If we assume that the phase errors are independent and identically distributed and define $\beta = < exp[-j\phi_i] >$, for $i = 1, \ldots, N$, then the expectation of (18) results in

$$<B\mathbf{a}\mathbf{a}^H B^H> = |\beta|^2\mathbf{a}\mathbf{a}^H + (1 - |\beta|^2)I \tag{19}$$

Since the magnitude of the complex exponential does not exceed unity, we certainly have $0 \leq |\beta| \leq 1$. Also, β=1 when the phase errors are identically zero. Evaluating β requires additional assumptions regarding the probability distribution of the phase errors or sensor element position errors. Analysis to this end is presented later in this paper.

Using (19) in (17), and using (17) in (16) gives

$$< b(t)'b^*(t)' > = |\beta|^2 \sum_{j=1}^{M} p_j \|\mathbf{w}^H\mathbf{a}_j\|^2 + (1 - |\beta|^2)\mathbf{w}^H\mathbf{w}\sum_{j=1}^{M} p_j + \mathbf{w}^H Q'\mathbf{w} \tag{20}$$

for the beamformer power output from the distorted array. Comparison of (7) and (20) reveals the

effect of array distortion on the power output: the nominal power output due to the M sources is attenuated by $|\beta|^2$, and a constant "background" power level (this terminology is due to Gilbert and Morgan in the reference previously cited) indistinguishable from uncorrelated noise is added, as represented by the second term of (20). Also, the ambient noise contribution is altered in some unknown way, presumably by a small amount, which is a valid assumption if the statistics of $n(t)'$ differ little from $n(t)$.

When sensor element position information is perfect, the value of β is unity and so the second term of (20) drops out; when unknown array distortion is present, the background power level (i.e., the second term of (20)) grows, and the signal power output (i.e., the first term of (20)) decreases as $|\beta|^2$. Eventually the second term of (20) dominates over the first (i.e., the white, background power dominates the signal power), and since this second term has no directional dependence, the beamformer becomes useless. The magnitude of β (and thus of the sensor element position errors) for which the background power term dominates over the signal power term also depends directly on the factor $w^H w$.

Gilbert and Morgan (in the previously cited reference) defined a function $K(w)$, applicable to the single source case, which in the present notation is

$$K(w) = w^H w/(w^H a_M)^2 \tag{21}$$

to serve as a measure of the beampattern susceptibility to degradation, i.e., the sensitivity, to random errors in the element positions (and, although not formally part of the present analysis, to the excitation coefficient magnitudes). The RHS of equation (21) is akin to the reciprocal of the directivity index because the numerator is proportional to what would be the total power output of an omni-directional sensor array of power output equal to the actual array, and the denominator is proportional to the power output due to a plane wave signal for the directionally discriminating array. Thus, large $K(w)$ indicates a lack of directional sensitivity.

One may easily prove that $K(w)$ is bounded from below by the reciprocal of the number of sensor elements by expressing w as the product of some scalar, say κ, and a unit vector, say $\mathfrak{W}$, so that

$$K(w)=K(\kappa\mathfrak{W})=\kappa^2/(\kappa\mathfrak{W}^H a_M)^2 \geq 1/|a_M|^2=1/N$$

Thus, beamformers for which $K(w)$ equals $1/N$ are the most robust with respect to random element position errors (and, incidentally, to errors in the excitation coefficient magnitudes). If w were con-

strained to unit magnitude, the minimum of $K(\boldsymbol{w})$ would occur ***only*** when $w_i{=}a_{iM}$, for $i{=}1,\ldots,N$, that is, when each component of the beamformer weight vector has the same magnitude, and the beamformer "points" in the direction of the DOA vector of interest.

With this interpretation, certain desirable beamformer characteristics become evident. Noting that $\boldsymbol{a}_M$ is fixed in magnitude and that $\boldsymbol{w}$ is not, the susceptibility to beampattern degradation is reduced for beamformers which have small a total power output, that is small magnitude of each sensor weight w_i, i=1,...,N, and strong correlation with the DOA of interest. For a beamformer searching various directions, the denominator of (21) will be small until the direction corresponding to source M is interrogated. Thus a small value of $K(\boldsymbol{w})$ would reduce the possibility of a false alarm by keeping the second term on the RHS of (20) small when the beamformer is not trained on the target. The next two sections make these desirable beamformer characteristics mathematically precise and present a means for deriving such a desirable beamformer.

BEAMPATTERN CONSTRAINTS

Suppose that the first M-1 sources emit interfering signals, and that only the Mth signal is of interest. A beamformer $\boldsymbol{w}$ may be designed so as to satisfy the following constraints:

$$b_j{=}\boldsymbol{w}^H\boldsymbol{a}_j{=}0,\text{ for }j{=}1,\ldots,M-1 \tag{22a}$$

$$b_M = \boldsymbol{w}^H\boldsymbol{a}_M{=}1 \tag{22b}$$

Constraints (22a) result in nullifying the beamformer output due to interfering sources 1 through $M-1$, and constraint (22b) serves both to normalize the magnitude of the array response and to steer the MRA toward the signal of interest. Equation (22b) also forces the denominator of (21) to unity. Note that (22) represents a set of M complex constraints (the equations) on N complex numbers (the entries of vector $\boldsymbol{w}$), so that M must be less than or equal to N in order to have a unique solution. The constraint equations (22) may be written in matrix form by defining the NxM matrix A of DOA vectors and the Mx1 vector $\boldsymbol{b}{=}[0,\ldots,0,1]^{\mathrm{T}}$. This convention is adopted in the following sections.

OPTIMUM LINEARLY-CONSTRAINED BEAMFORMING

The optimization criterion selected for this analysis is the minimization of the beampattern sensitivity to array distortion for a beamformer steered to the signal of interest. Thus the problem of determining the linearly constrained, optimum beamformer $\boldsymbol{w}_o$ is stated mathematically as

$$\min_{\boldsymbol{w}} K(\boldsymbol{w}) = K(\boldsymbol{w}_o) \tag{23a}$$

$$\text{subject to } A^H \boldsymbol{w} = \boldsymbol{b} \tag{23b}$$

The solution to (23) may be found by the method of Lagrange multipliers. Accordingly we define the Mx1 vector $\boldsymbol{v}$ of undetermined Lagrange multipliers and write

$$H(\boldsymbol{w}) = K(\boldsymbol{w}) + \boldsymbol{v}^H(A^H \boldsymbol{w} - \boldsymbol{b})$$

Setting equal to zero the gradient of $H(\boldsymbol{w})$ with respect to $\boldsymbol{w}$ gives the solution to (23) as

$$\boldsymbol{w}_o = -(1/2)A\boldsymbol{v} \tag{24}$$

The Lagrange multipliers $\boldsymbol{v}$ are determined by applying the constraints (23b) to give

$$A^H \boldsymbol{w}_o = \boldsymbol{b} = -(1/2)A^H A\boldsymbol{v}$$

which may be solved for

$$\boldsymbol{v} = -2(A^H A)^{-1}\boldsymbol{b} \tag{25}$$

Note that the matrix inverse $(A^H A)^{-1}$ in (25) exists if and only if the M columns of A (i.e., the DOA vectors corresponding to the M signals) are linearly independent. This imposes certain conditions on the array geometry equivalent, for example, for the single-line, uniformly spaced sensor array, to insuring that the sensors are less than one-half wavelength apart, or more generally, that directional aliasing of the received signals does not occur.

Substituting (25) into (24) gives

$$\boldsymbol{w}_O = A(A^H A)^{-1}\boldsymbol{b} \tag{26}$$

as an explicit expression for the optimum, linearly-constrained beamformer. Again, $\boldsymbol{w}_O$ is optimum in that the beampattern sensitivity to array distortion is minimized, in the sense of Gilbert and Morgan, expressed here as (23a). Substitution of (26) in (23b) shows that the constraints are indeed satisfied by $\boldsymbol{w}_O$.

Using (26) in (20) gives the power output of the optimum, linearly-constrained beamformer as

$$< b(t)'b^*(t)' > = |\beta|^2 p_M + (1 - |\beta|^2)K(\boldsymbol{w}_o)\sum_{j=1}^{M} p_j + \boldsymbol{w}_o Q' \boldsymbol{w}_o \tag{27}$$

where the constraints (23b) have been applied to set $\boldsymbol{w}_O{}^H \boldsymbol{a}_j = 0$ for $j=1,\ldots,M-1$ and $\boldsymbol{w}_O{}^H \boldsymbol{a}_M = 1$. In the next section we examine the ability of the beamformer $\boldsymbol{w}_o$ to reject interfering signals.

NULL-STEERING PERFORMANCE

The beampattern of a beamformer is defined as the beamformer output power, as a function of incident, planewave signal propagation direction, for a single, unit-power source. Let $\boldsymbol{u}=[u_x,u_y,u_z]^T$ represent the direction cosines of a vector pointing in the direction of propagation, let $\boldsymbol{a}$ represent the corresponding DOA vector for the undistorted array, and let B represent the corresponding diagonal matrix of complex exponential of phase errors for the distorted array. The beampattern $\mathfrak{B}(\boldsymbol{u})$ is then determined by selecting a suitable set of incident directions, calculating the corresponding direction cosines, the DOA vector, and the matrix B, and then calculating the expected value of the beamformer output $\mathfrak{B}(\boldsymbol{u}) = < |\boldsymbol{w}_O{}^H B\boldsymbol{a}|^2 >$ for each direction. Using (19) to evaluate the expectation gives

$$\mathfrak{B}(\boldsymbol{u}) = \boldsymbol{w}_O{}^H < B\boldsymbol{a}\boldsymbol{a}^H B^H > \boldsymbol{w}_O = |\beta|^2 \boldsymbol{w}_o{}^H \boldsymbol{a}\boldsymbol{a}^H \boldsymbol{w}_o + (1 - |\beta|^2)K(\boldsymbol{w}_o) \tag{28}$$

Of particular interest is when the beampattern is evaluated for $\mathbf{u}=\boldsymbol{u}_j$, for $j=1,\ldots,M$, i.e., in the directions of the interfering and the desired signals. The beampattern level (28) evaluates at these points as

$$\mathfrak{B}(\boldsymbol{u}_j) = < |\boldsymbol{w}_O{}^H B_j \boldsymbol{a}_j|^2 > = (1 - |\beta|^2)K(\boldsymbol{w}_O), \text{ for } j=1,\ldots,M-1 \tag{29a}$$

$$\mathfrak{B}(u_M) = <|w_o{}^H B_M a_M|^2> = |\beta|^2 + (1-|\beta|^2)K(w_o) \tag{29b}$$

By forming the ratio of (29a) to (29b), a convenient measure is obtained for assessing the effectiveness of a beamformer to reject interfering signals relative to the reception of a signal of interest. This ratio may be referred to as the output interference-to-signal ratio (OISR) and is written

$$\text{OISR} = \frac{\mathfrak{B}(u_j)}{\mathfrak{B}(u_M)} = \frac{(1-|\beta|^2)K(w_o)}{|\beta|^2 + (1-|\beta|^2)K(w_o)} \tag{30}$$

(Some readers may prefer to think in terms of signal-to-interference ratios, but then the possibility of unbounded values must be handled for the case of perfect nulls. Equation (30) is more convenient as written since it remains bounded.) Equation (30) is plotted in figure 1 for five values of $K(w_o)$. The increasing sensitivity to array distortion with increasing values of $K(w_o)$ is evident: for small values of $K(w_o)$, the OISR remains smaller than for larger values of $K(w_o)$ as $|\beta|^2$ decreases. This implies that the *possibility* of decreased beamformer sensitivity to sensor position errors arises as the number of sensor elements is increased since, as the number of sensor elements N is increased, $K(w_o)$ may decrease (recall that $K(w_o) \geq N^{-1}$).

An upper bound on the OISR is obtained by determining

$$max\{\text{OISR}\} = \frac{\underset{1 \leq i \leq N}{max}\{|w_o{}^H B_j a_j|^2\}}{\underset{1 \leq i \leq N}{min}\{|w_o{}^H B_M a_M|^2\}} \tag{31}$$

When the actual array element positions correspond to the assumed element positions (i.e., α_i=0 for i=1,…,N), B_j is the identity matrix for j=1,…,M and so (31) yields zero. Since $|w_o{}^H B_j a_j|^2$ is non-negative, the effect of pre-multiplying a_j by B_j for cases in which the actual array element positions differ from the assumed positions is to increase (or perhaps leave unchanged) from zero the numerator of (31). Pre-multiplying a_M by B_M may decrease or increase (or leave unchanged) the denominator. These assertions can be seen by the following asymptotic analysis where we drop the subscript j, denoting particular signal directions, for notational convenience. Expanding B in a Taylor series about zero gives

$$B = I - j\Phi - \frac{1}{2}\Phi^2 \tag{32}$$

where $\Phi = diag\{\phi_1, \ldots, \phi_N\}$ is the diagonal matrix with entries representing the signal phase delay deviations, for any possible signal propagation direction, due to sensor element position errors. Equation (32) may be approximated by retaining only terms linear in Φ so long as the following condition is satisfied:

$$\tfrac{1}{2} L_p[\Phi^2] \ll L_p[\Phi] \tag{33}$$

where $L_p[*]$ denotes any of the matrix p-norms. For a brief summary of matrix norms, refer to the appendix; for intuitive understanding, think of the matrices as scalars and the p-norms as absolute values. Equation (33) may be alternatively stated as the requirement that the error due to truncating (32) beyond the linear term be much less than the array element phase errors due to position errors. Applying the first of the p-norms, i.e., the maximum absolute column sum, gives

$$L_1[\Phi] = \max_i |\phi_i| \tag{34}$$

and

$$L_1[\Phi^2] = \max_i |\phi_i^2| \tag{35}$$

Using (34) and (35), and since the ϕ_i are real numbers, it is clear that (33) is satisfied for

$$\max_i |\phi_i| \ll 2 \tag{36}$$

If (36) is true, then it must be true that $|\phi_i| \ll 2$ for all $i=1,\ldots,N$. Equation (36) may be satisfied by imposition of a requirement that $\frac{\omega}{c}\max_i\{|\alpha_i|\} \ll 2$, or that

$$\max_i\{|\alpha_i|\} \ll \frac{2c}{\omega} \tag{37}$$

since $\phi_i = \frac{\omega}{c} \cdot u^T \alpha_i$. and $|u^T\alpha_i| \leq |u|\,|\alpha_i| = |\alpha_i|$, with u a unit vector. From (37) it is clear that the maximum sensor position error should be much less than approximately one-third wavelength to justify truncating (32) beyond the linear term.

Using the linear approximation to (32) we may write

$$\boldsymbol{w_o}^H B\boldsymbol{a} \sim \boldsymbol{w_o}^H[I - j\Phi]\boldsymbol{a}$$

The general form of both the numerator and denominator of (31) is then given approximately by

$$\mathfrak{B}(\boldsymbol{u}) = < |\boldsymbol{w_o}^H B\mathbf{a}|^2 > \sim [\boldsymbol{w_o}^H \mathbf{a}|^2 + 2Im\{\boldsymbol{w_o}^H \Phi\mathbf{a}\mathbf{a}^H \boldsymbol{w_o}\} + |\boldsymbol{w_o}^H \Phi\mathbf{a}|^2] \tag{38}$$

The right-hand side of equation (38) evaluated for specific propagation directions $\boldsymbol{u}_j$ for j=1,...,M, gives

$$\mathfrak{B}(\boldsymbol{u}_j) \sim |\boldsymbol{w_o}^H \Phi\boldsymbol{a}_j|^2, \text{ for } j=1,\ldots,M-1 \tag{39a}$$

and

$$\mathfrak{B}(\boldsymbol{u}_M) \sim [1 + 2Im\{\boldsymbol{w_o}^H \Phi_M \boldsymbol{a}_M\} + |\boldsymbol{w_o}^H \Phi_M \boldsymbol{a}_M|^2] \tag{39b}$$

since $\boldsymbol{w_o}^H \boldsymbol{a}_j=0$, for j=1,...,M-1, and $\boldsymbol{w_o}^H \boldsymbol{a}_M=1$. Equation (39a) gives the approximate beampattern level for the direction of steered nulls, and (39b) gives the approximate level for the direction of the steered MRA when the sensor elements have been displaced from assumed or measured positions by a small amount. Note that the second term in (39b) may assume either negative or positive values depending on the element position errors.

Using (39) in (31) and carrying out the minimization in the denominator provides an upper bound on the beamformer ability to reject an undesired interference relative to the desired signal. This ratio is given by

$$max\{\text{OISR}\} \sim \frac{\max_{1 \le i \le N} |\boldsymbol{w_o}^H \Phi_j \boldsymbol{a}_j|^2}{1 - 2|\max_{1 \le i \le N} Im\{\boldsymbol{w_o}^H \Phi_M \boldsymbol{a}_M\}| + |\boldsymbol{w_o}^H \Phi_M \boldsymbol{a}_M|^2} \tag{40}$$

Equation (40) can be made more explicit by first using properties of p-norms to write

$$|\boldsymbol{w_o}^H \Phi\boldsymbol{a}_j| \le |\boldsymbol{w_o}|\,|\Phi_j \boldsymbol{a}_j| \le |\boldsymbol{w_o}|\,|\boldsymbol{a}_j|\,L_2[\Phi_j], \text{ for } j=1,\ldots,M, \tag{41}$$

noting that in this case we have

$$L_2[\Phi_j] = L_1[\Phi_j] = \frac{\omega}{c}\max_i\{|\alpha_i|\}, \tag{42}$$

and that $|\boldsymbol{a}_j|^2=N$, and then substituting $|\boldsymbol{w_o}|^2=K(\boldsymbol{w_o})$. The first inequality in (41) is the Cauchy-

Schwartz inequality; the second is a property of p-norms in general. Equation (42) is due to the fact that the Φ_j are diagonal matrices. Note that the vector norms on the right-hand side of (41) are constants for a given beamformer and signal environment, and that (42) expresses the size of sensor position measurement error in radian wavelengths, i.e., the RHS of (42) is equal to $2 \cdot \pi$ radians per wavelength times the maximum position error.

With these steps, equation (40) becomes

$$max\{\mathrm{OISR}\} \sim \frac{N \cdot K(w_o) \cdot \left(\frac{\omega}{c} \max_i\{\,|\alpha_i|\,\}\right)^2}{1 - 2(N \cdot K(w_o))^{1/2} \cdot \frac{\omega}{c} \max_i\{\,|\alpha_i|\,\} + N \cdot K(w_o) \cdot \left(\frac{\omega}{c} \max_i\{\,|\alpha_i|\,\}\right)^2} \tag{43}$$

Equation (43) offers some insight into assessing beamformer performance degradation due to sensor element position errors: The maximum OISR is bounded from above by a function that, for small arguments, is dominated by a quadratic term in a variable directly proportional to the sensor element position errors. Figure 2 depicts the upper bound.

STATISTICAL ANALYSIS OF PHASE ERRORS

In order to make practical use of (30) or of figure 1, a definite relationship must be found between the parameter β and some physical quantity related to array distortion. This requires knowledge, or perhaps a clever choice of assumptions, regarding the probability density function of the sensor element displacements α_i, for i=1,...,N. Recall that in deriving (19) the phase errors were assumed to be independent and identically distributed. Therefore, the evaluation need be done for only one arbitrary spatial location.

Let α,=$[\alpha_x,\alpha_y,\alpha_z]$ denote the Cartesian components of any sensor position error, and let $p(\alpha)=p(\alpha_x,\alpha_y,\alpha_z)$ denote the probability density function (pdf) or these errors. (Note the notational distinction between the symbol $p(\cdot)$ suffixed with an explicit argument, denoting a pdf, and the symbol p_j carrying a subscript j, used earlier denoting variance or signal power.) The definition of expectation gives

$$\beta = \int_{-\infty}^{\infty} exp[-j \tfrac{\omega}{c} \mathbf{u}^T \alpha]\; p(\alpha_x,\alpha_y,\alpha_z) d\alpha_x d\alpha_y d\alpha_z \tag{44}$$

If we assume independent random variations of α_x, α_y, and α_z, then

$$p(\alpha_x,\alpha_y,\alpha_z)=p(\alpha_x)p(\alpha_y)p(\alpha_z),$$

and the integral (44) is separable.

If we now assume the component pdf's to be Gaussian with means μ_x, μ_y, and μ_z, and standard deviations σ_x, σ_y, and σ_z, respectively, the expectation[4] in (44) gives

$$\beta=h(\omega u_x/c)\ h(\omega u_y/c)\ h(\omega u_z/c) \tag{45}$$

where, for any of the Cartesian components u,

$$h(\omega u/c)=exp\left(-\tfrac{1}{2}(\tfrac{\sigma\omega u}{c})^2-\tfrac{j\omega u\mu}{c}\right) \tag{46}$$

so that

$$|\beta|^2=exp\left\{-\left(\tfrac{\omega}{c}\right)^2\cdot\left(\sigma_x{}^2u_x{}^2+\sigma_y{}^2u_y{}^2+\sigma_z{}^2u_z{}^2\right)\right\} \tag{47}$$

If the additional assumption is made that the x-, y-, and z-direction variances are equal so that the sensor position errors are spherically symmetric, then equation (47) becomes

$$|\beta|^2=exp\left\{-\left(\tfrac{\sigma\omega}{c}\right)^2\right\} \tag{48}$$

where $\sigma=\sigma_x=\sigma_y=\sigma_z$, by assumption, and $u_x{}^2+u_y{}^2+u_z{}^2=1$, by definition. Equations (47) and (48) confirm the earlier comments about $|\beta|^2=1$ implying perfect sensor position information and explicitly indicates that $|\beta|^2$ decreases as unknown array distortion (as measured by variance of position errors) increases.

CONSEQUENCES OF THE ANALYSIS

Equation (46) indicates that if the signals propagate largely in a particular plane so that one of the direction cosines is small, then the sensor position error variance corresponding to that direction

cosine has little effect on beamformer performance. That is $u \ll 1$ implies $h(\omega u/c) \sim 1$. This fact can be used to tactical advantage by the sonar operator if it is known that random array motion is larger in one particular direction than in others. The effect of the sensor position error variance could be minimized by so aligning the array that signals propagate perpendicular to that component of random array motion, thereby making the corresponding direction cosine small.

Equation (47) indicates that the mean sensor position error (for individual sensors, not the average over all sensors) has no influence on the value of $|\beta|^2$, and thus neither on the OISR. This is consonant with intuition because one can readily reason that a whole-body translation of the array, i.e., identical, rectilinear displacement of all elements, in a signal field of planar waves is not detectable. (If this were untrue, the whole concept of towed-array sonar and other applications of mobile-platform-mounted sensor arrays would be misguided because the act of towing an array of sensors imparts just such a whole-body translation.)

Equation (48) provides a link associating OISR and beampattern degradation to physically meaningful quantities, specifically (48) relates $|\beta|^2$, and thus OISR, to sensor element position error standard deviation. In (48), this standard deviation is expressed in dimensionless form, i.e., the parameter $\frac{\sigma\omega}{c}$ expresses the position error standard deviation relative to the signal wavelength. This implies that the effects of array distortion can be ameliorated by operating the sensor array at lower signal frequencies (i.e., longer wavelengths) as well as by more effectively controlling or measuring sensor element positions (ideas which despite the analysis may be intuitively obvious, but which here are explicit and quantifiable). Alternatively, this implies that operation at higher acoustic frequencies *requires* improved sensor position control or measurement capabilities.

Figures 3a and 3b present the OISR verses the dimensionless array distortion parameter $\frac{\sigma\omega}{c}$. As in figure 1, the increasing sensitivity to array distortion with increasing $K(\mathbf{w})$ is evident. Note that figure 3 does not simply repeat on different axes the information presented in figure 1: it displays a particular relationship between array distortion and null-steering performance. While the ordinate is simply the OISR of figure 1, the parameter $|\beta|^2$ on the abscissa has been transformed to the dimensionless array distortion parameter $\frac{\sigma\omega}{c}$ using the particular relationship given in (48). This relationship depends on the choice of the pdf assumed representative of array element motion. Figure 1 is more general but requires evaluation of β in terms of the sensor element position probability distribution.

Figure 3a may be used practically to specify the maximum allowable sensor position error

standard deviation for a given beamformer after selecting the desired amount of interference rejection: An acceptable OISR is selected and noted on the ordinate, and then the value of $\frac{\sigma\omega}{c}$, for the appropriate value of $K(\boldsymbol{w})$ which applies to the beamformer, is found on the abscissa. For example, with $K(\boldsymbol{w})$=0.01, a "null" 30 dB below the MRA requires sensor element position error less than about $\frac{\sigma\omega}{c}$=0.3 which is equivalent to standard deviation σ<0.05 wavelengths. Thus obtaining deep nulls apparently presents a very demanding position error tolerance requirement.

Figure 3b repeats on a linear scale the information of figure 3a. The degree of sensitivity to array distortion seems more apparent on this plot than in figure 3a. For the largest value of $K(\boldsymbol{w})$ plotted, the OISR rapidly increases from zero as array distortion increases from zero; for the smallest value of $K(\boldsymbol{w})$ plotted, greater tolerance to array distortion is revealed by the OISR remaining relatively small until the array distortion parameter reaches approximately one-third.

SOME COMMENTS ON THE ASSUMPTIONS

Analytic studies regarding towed flexible cylinders have shown (see for instance Kuo[5] and the references therein), and intuition suggests, that neither motion of individual sensor elements nor the directional components of motion for a single sensor element are independent in towed sonar arrays. As with a flapping flag or a child's jumprope, motion at one point on the body containing the sensors is coupled to the motion at other points. The coupling depends (at least) on the stiffness or rigidity of, and the tension supported by, the material forming the mechanical coupling and (perhaps) on the medium in which the array is immersed. Barring this assumption strains the preceding analysis by invalidating (19). Instead of being able to factor out the parameter $|\beta|^2$ from each entry, each entry of matrix $\boldsymbol{aa}^H$ is individually scaled by a (possibly different) factor dependent on the cross correlation between the element phase errors.

For the particular application to conventional towed array sonar, a spherically symmetric distribution is intuitively unrealistic because of the nature of the mechanical connection between sensor elements, the elements being coupled by, for example, a section of flexible hose for sensors on a single line, or indirectly through either a headline or some type of (practically) rigid structure between lines. However, for non-symmetric, independent, Gaussian distributions we may *define*

$$\sigma^2=\sigma_x^2u_x^2+\sigma_y^2u_y^2+\sigma_z^2u_z^2$$

so that (48) remains applicable.

The assumption of identical distributions for each sensor element is also flawed since the tension due to hydrodynamic drag, which is maximum in a towed line near the point of attachment to the towing structure, approaches zero near the free end and thus affects sensor motion differently for different positions in the array. Barring this assumption has an effect similar to barring the assumption of independent phase errors in that a single scalar constant can no longer be factored out of each entry in the matrix $\mathbf{aa}^H$, but instead each entry has a (possibly) unique multiplicative scale that depends on the distribution pertaining to each of the two spatial locations represented by each matrix entry.

The assumption of a Gaussian distribution may perhaps be more appropriately replaced by a sine wave probability distribution[6] since, at least for motion transverse to the axis of a towed, flexible cylinder, the dynamic behavior of any point on said cylinder does exhibit oscillations (although these are not strictly periodic). If the simplicity of integration afforded by the assumption of Gaussian statistics is absent, it may be possible to estimate β by bringing to bear the arsenal of asymptotic analysis.

Admittedly, assuming the position errors to be independent, identically-distributed Gaussian random variables may not represent reality, but these *assumptions do allow us to make quantitative*, albeit qualified, statements about array performance degradation, as well as make the problem tractable. Experimental investigation (i.e., measurement and statistical analysis) of sensor motion is required to validate or assist in modifying the assumptions regarding sensor element position errors.

COMPUTER SIMULATION OF NULL-STEERING DEGRADATION

The following sections present results of a computer simulation of null-steering performance degradation. The simulation does not accurately represent the preceding analysis because the information used is not random, but rather each case represents one particular realization of possible array output.

For this study, the α_i are taken from interpolated, experimental data from previous at-sea testing[7] and are multiplicatively scaled to simulate increasing *deviations from the* assumed element *positions*. Signals from pingers embedded in rope drogues were acoustically tracked to produce estimates of the locations of up to six points on each of up to six lines. For this computer exercise, a numerical algorithm[8] implementing a solution by the method of characteristics to the dynamic equations of motion[9] for towed flexible cylinders in various configurations was used to interpolate addi-

tional, plausible points on the lines hypothesized to be sensor locations. One-hundred and sixty-two such hypothetical sensor locations were generated for each of six lines in a vertical plane, and these points were spaced at approximately 1/3 wavelength along each line for a 1 kHz signal of interest.

The signal of interest was assumed broadside to the baseline array, and a single null was placed in the plane of the vertical baseline array, 17° above the horizontal, so as to simulate nulling of direct path propagation from a towing ship.

For each set of assumed sensor positions the optimum, linearly-constrained beamformer $\mathbf{w}_o$ as well as $K(\mathbf{w}_o)$ were computed; for each set of deviations in sensor positions, the OISR and the variance of the sensor position deviations were computed. The computed OISR were plotted against the computed standard deviation, expressed in dimensionless form using the parameter $\frac{\sigma\omega}{c}$, of the sensor element position errors and compared with the predicted OISR, given by (30), for the minimum-sensitivity beamformer (i.e., the value of $K(\mathbf{w})$ is set equal to $1/N$ for the predictions). The computed sensitivity for the beamformer in fact was equal to $1/N$ within computer roundoff error. Note that while the variance of any particular set of position errors is not necessarily the same as the expected value (i.e., the ensemble average) of the squared deviation for a sensor, as used in the preceding statistical analysis, the previous assumptions regarding the statistical properties of the position errors along with the additional assumption of stationarity imply ergoticity and thus that the sample standard deviation provides an unbiased estimate of the standard deviation determined by ensemble averaging.

Figure 4 shows the predicted OISR and the computed OISR for three realizations or snap-shots of a three-line array; figure 5 shows a similar comparison for a six-line array. The arrays were planar in a vertical plane for the baseline case and were approximately co-planar for the two comparison cases. The figures indicate that the computed OISR resulting from the simulation is 15 dB to 25 dB below that predicted by (30) for all the configurations simulated. For array distortion less than about 1/3 wavelength, all curves representing the simulations appear to follow the shape of the predicted curve; for larger array distortion the baseline simulation (i.e., that simulation for which the undistorted arrays were planar) exhibit the most similarity in character to the theoretically predicted OISR. Simulations for which the undistorted array were at best near-planar produced OISR curves not tracking the predicted curves for array distortion parameter values larger than approximately three-quarter to one-and-one-quarter wavelengths.

SUMMARY

This paper has presented a study of the effect of array distortion on null-steering performance of a linearly constrained, minimum sensitivity beamformer. The analytical treatment used matrix notation to describe beamforming and to derive expressions for the signal and power outputs of an arbitrary beamformer (arbitrary as to the number of sensor elements and the configuration of those elements) subject to errors in sensor element position data (as obtain from assumption of some nominal shape or by imprecise measurement).

The expression for the power output was then simplified by assuming the sensor element position errors to be independent, identically-distributed random variables, thus revealing the effect of array distortion on different components of the output power and allowing identification of a measure of susceptibility to distortion. This measure of sensitivity was shown to depend primarily on the beamformer (i.e., on the norm of the sensor element weighting vector).

A derivation of a minimum-sensitivity beamformer was presented next. This beamformer also satisfied constraints on the signal (and thus on the power) output allowing placement of the MRA and of nulls in arbitrary directions for the undistorted array. A performance criterion was defined measuring the effectiveness of null steering, and an upper bound on this measure of performance was derived for small sensor position errors.

Further assumptions on the statistical properties of the sensor element position errors yielded an expression for the performance degradation as a function of the position error standard deviation. This expression was treated as a theoretical prediction and compared to results of a simplified computer simulation. This comparison indicates that the predicted performance degradation is conservative, i.e., degradation in performance is overestimated.

APPENDIX

The norms of vectors and matrices allow quantitative assessment of the notions of size and distance directly analogous to the familiar concepts of size or magnitude associated with the absolute value of real numbers, with the modulus of complex numbers, and with the euclidean concepts of size (i.e. magnitude) of vectors and distance between points in Cartesian space.[10] In fact, the absolute value of real numbers, the modulus of complex numbers, and the magnitude of vectors are each particular examples of the more general class of functions called norms.

A vector norm $f(x)$ associates a non-negative, real number with every (complex) Nx1 vector x and satisfies the following properties:

$f(x) \geq 0$ for any x, with the equality holding if and only if $x = 0$. (A-1)

$f(x + y) \leq f(x) + f(y)$ for all vectors x and y. (A-2)

$f(ax) = |a| f(x)$ for all scalars a (where the vertical bars denote absolute value). (A-3)

Matrix norms are defined the same way and satisfy the same criteria (A-1) through (A-3) as set forth above for vectors. A special class of norms called the Hölder or p-norms (denoted in this paper by $L_p[*]$) applied to MxN matrix A and NxR matrix B additionally satisfy

$$L_p[AB] < L_p[A]L_p[B] \tag{A-4}$$

that is, the p-norms applied to matrices possess a "multiplicative" triangle inequality in addition to the usual triangle inequality (A-2). The p-norms for vectors are defined by

$$L_p[x] = (|x_1|^p + \dots + |x_N|^p)^{1/p} \tag{A-5}$$

where the x_i, i=1,...,N denote the entries of the Nx1 vector x, and p is any positive integer. The p-norms used in this paper are L_1 and L_2 (with L_1 being recognized as the sum of the absolute values of the entries of x, and with L_2 being recognized as the familiar euclidean norm or magnitude of vector x).

The definition of the p-norms applied to matrices relies on equation (A-5) but is more complicated than warrants inclusion in this paper, however some significant and useful results are summarized below. The L_1 norm of an MxN matrix A is equal to the maximum absolute column sum, or

$$L_1[A] = \max_j \sum_{i=1}^{M} |a_{ij}| \tag{A-6}$$

The L_2 norm is bounded by

$$L_1[A]/M^{1/2} \leq L_2[A] \leq N^{1/2} L_1[A] \tag{A-7}$$

Most significantly, for purposes in this paper, when L_1 and L_2 are applied to diagonal matrices, both norms are equal to the absolute value of the largest entry. This last result leads directly to equations (34), (35) and (42) in the body of this paper.

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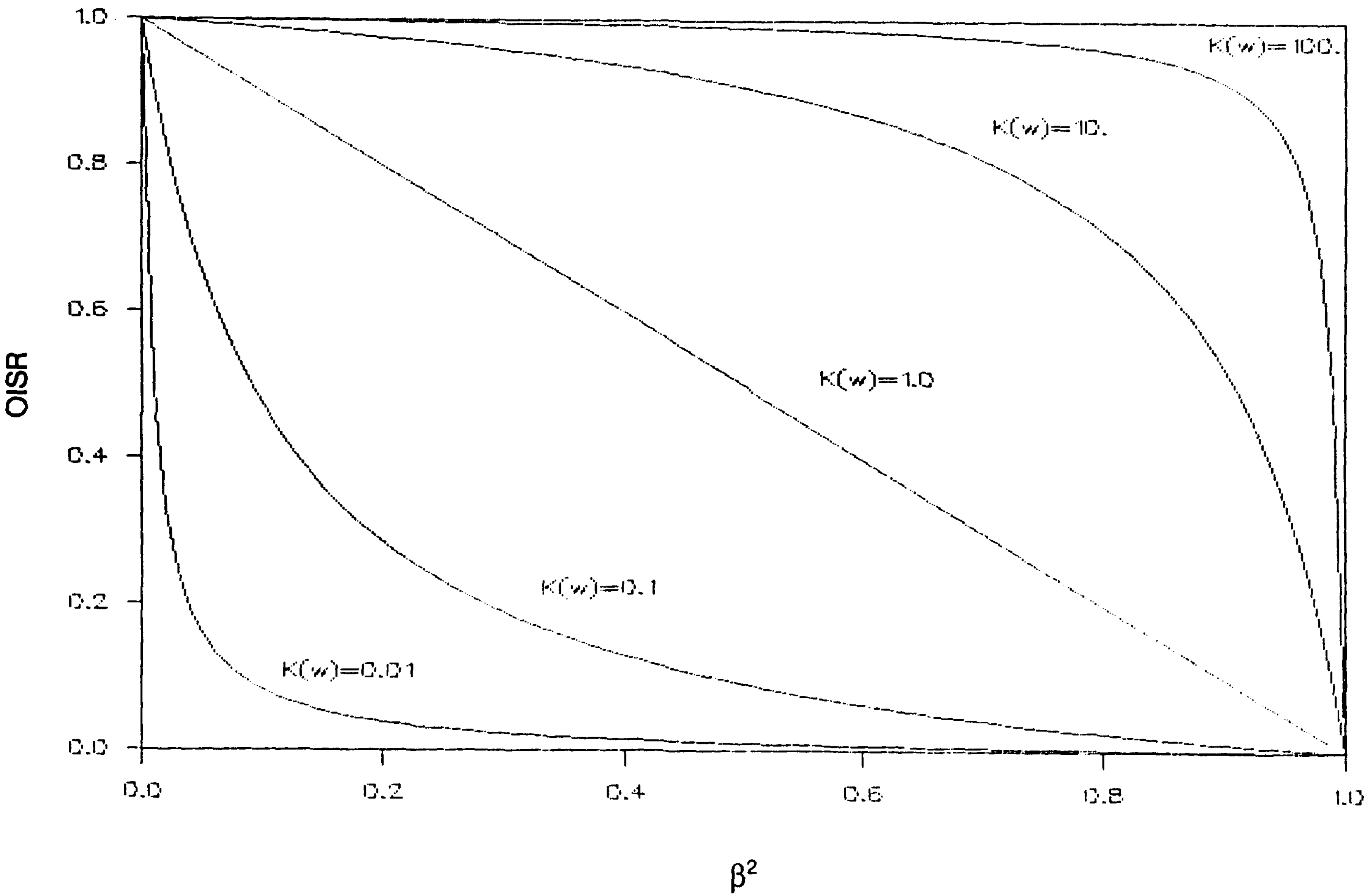

Figure 1. Output Interference-to-Signal Ratio (OISR) for Five K (w)

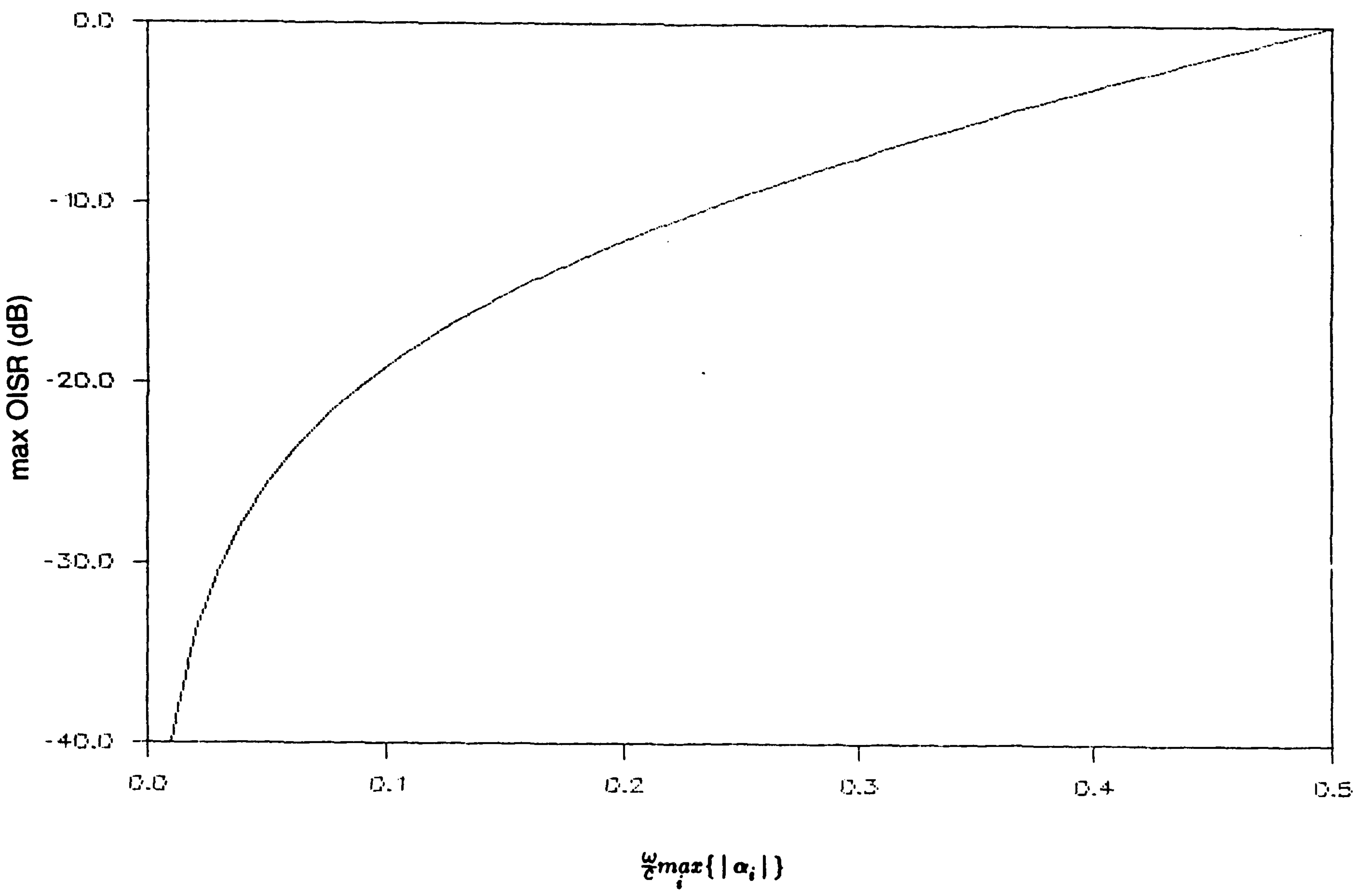

Figure 2. Upper Bound on Output Interference-to-Signal Ratio (OISR)

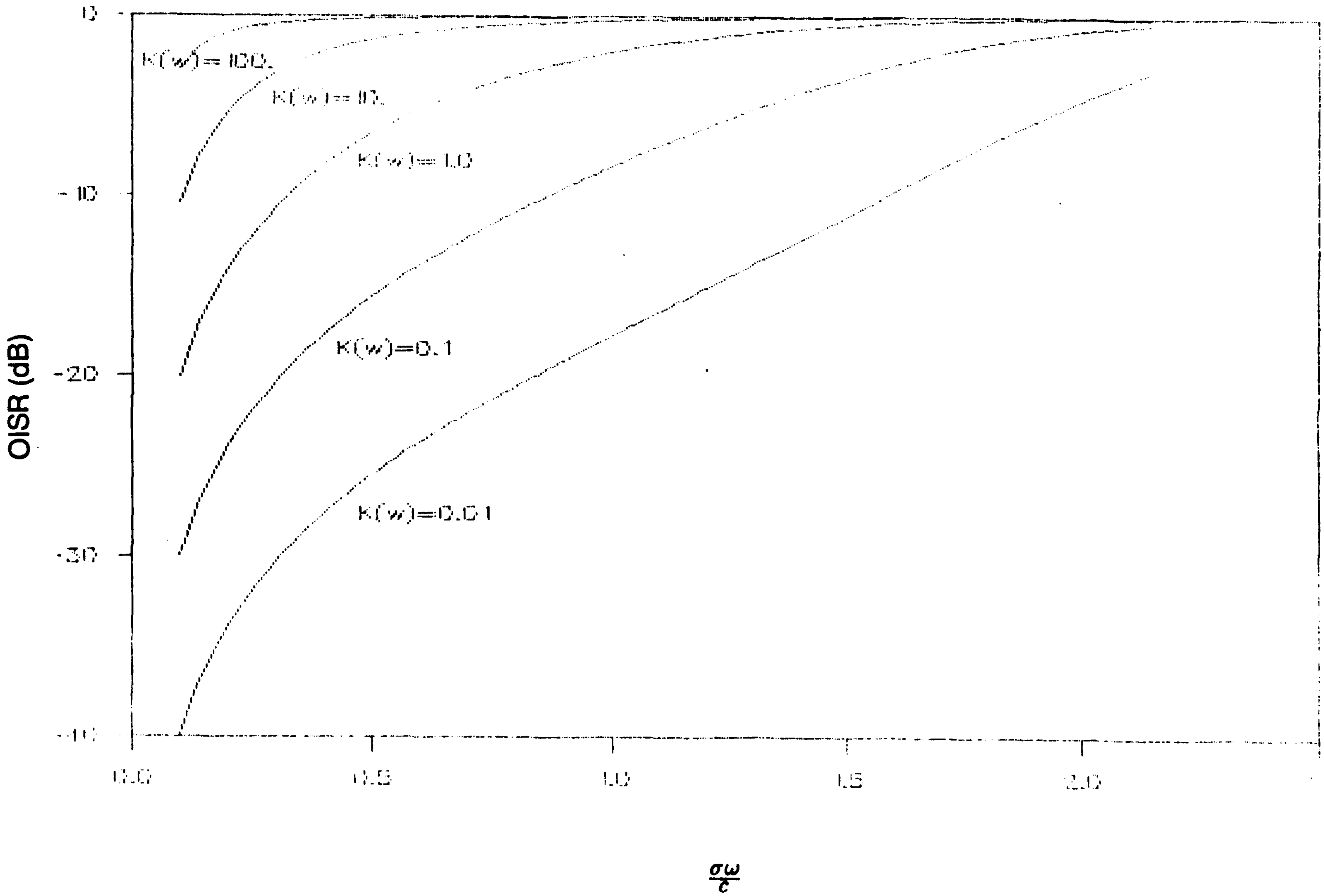

Figure 3a. Effect of Position Errors on Output Interference-to-Signal Ratio (OISR) in dB

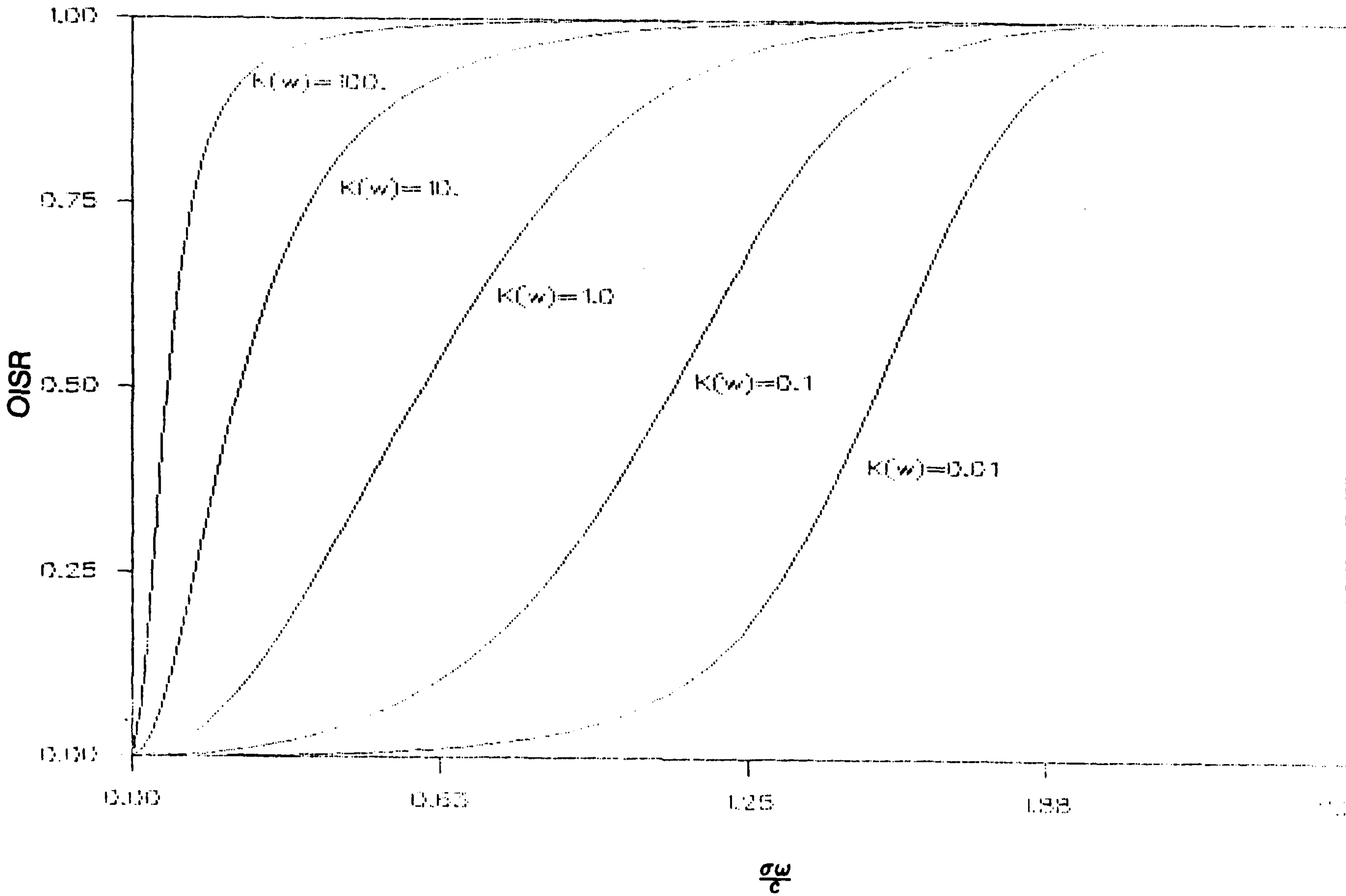

Figure 3b. Effect of Position Errors on Output Interference-to-Signal Ratio (OISR)

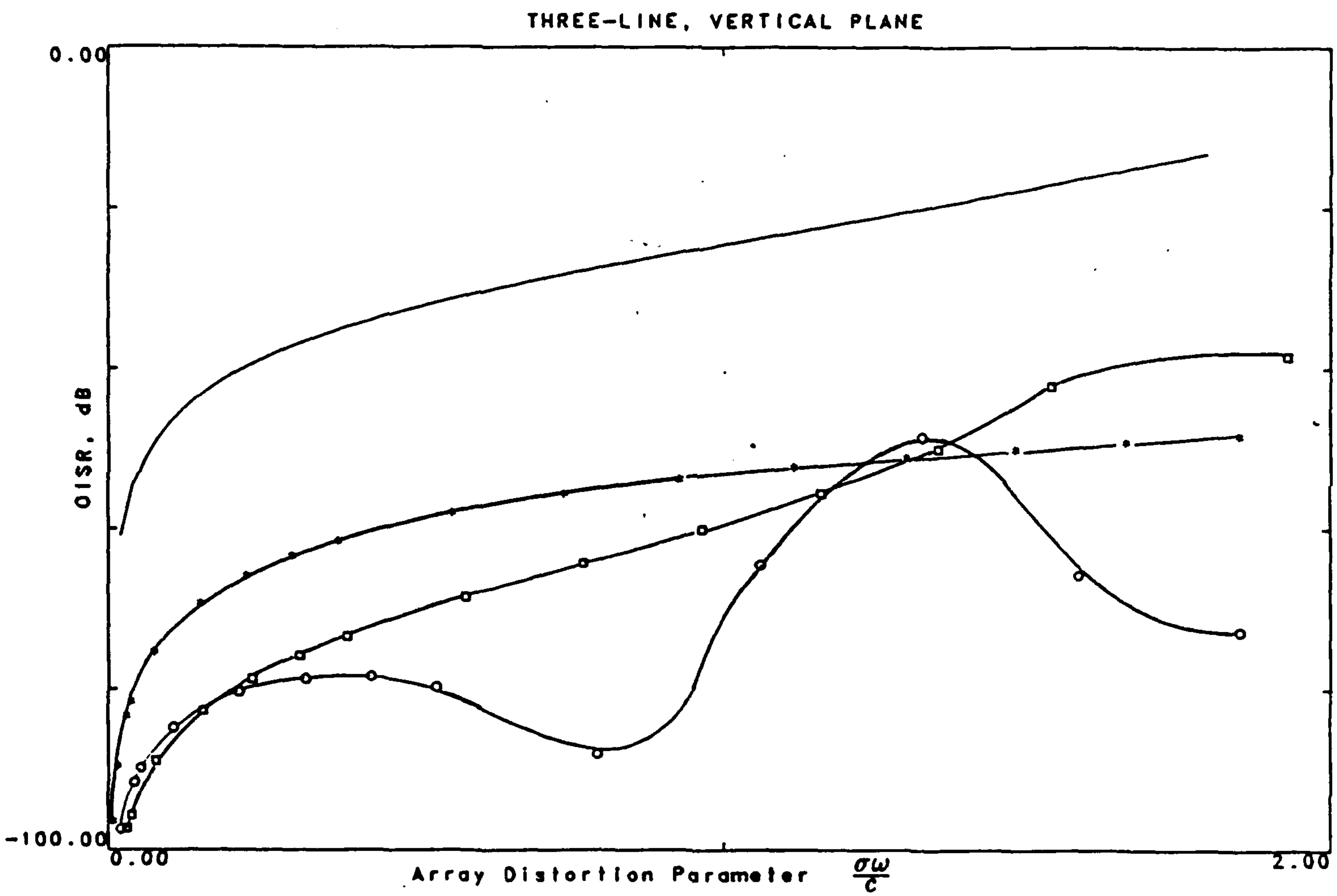

Figure 4. Computer Simulation for a Three-Line Array

— Theoretical Prediction
* Initial Period (Planar Referenced Array)
□ Later Period (Near Planar Referenced Array)
o Last Period (Near Planar Referenced Array)

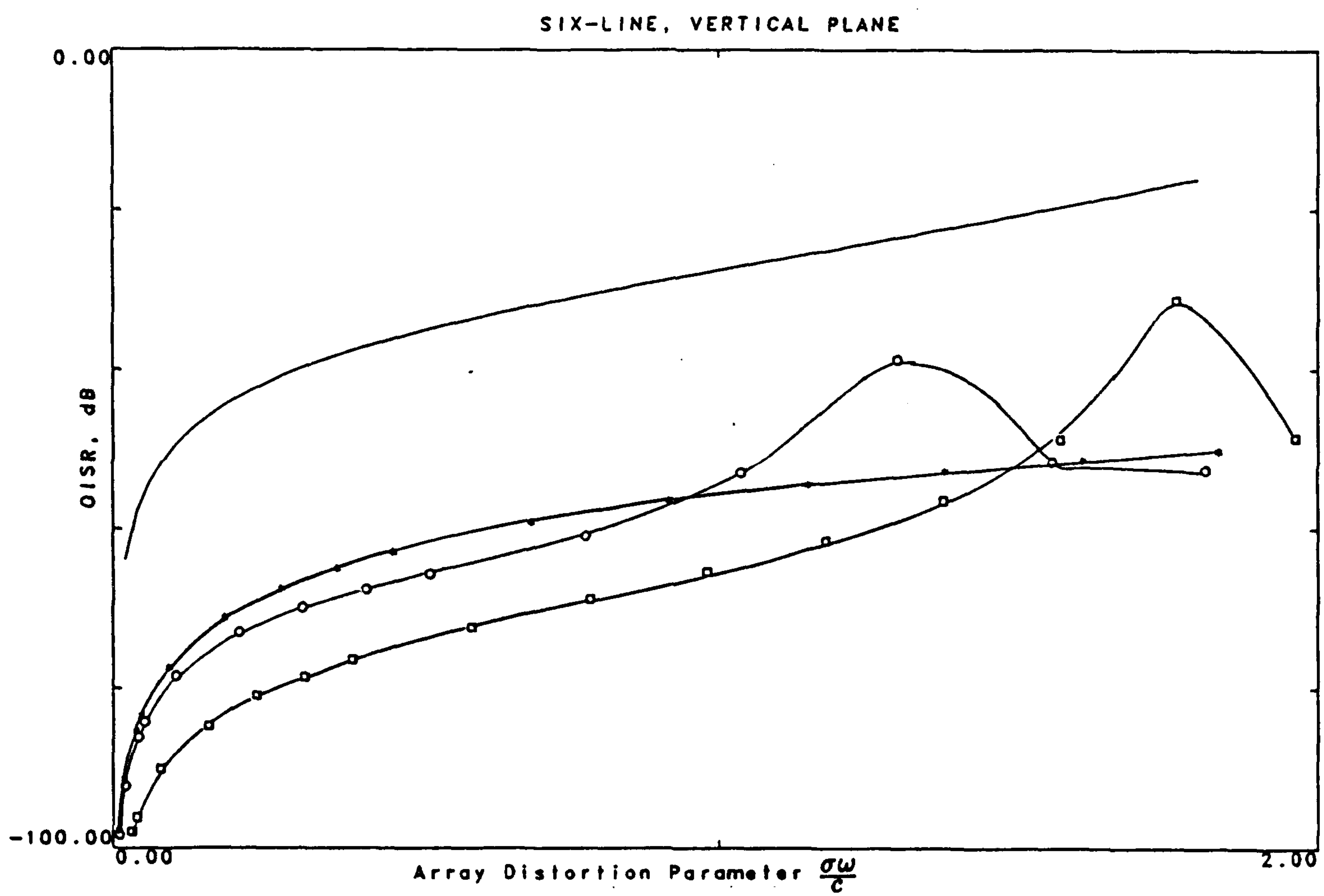

Figure 5. Computer Simulation for a Six-Line Array

— Theoretical Prediction

* Initial Period (Planar Referenced Array)

□ Later Period (Near Planar Referenced Array)

o Last Period (Near Planar Referenced Array)

NULL-STEERING PERFORMANCE DEGRADATION
DUE TO SENSOR ELEMENT POSITION
MEASUREMENT ERRORS

Date: 31 July 1989

Berend M. Tober

Towed Array and Ocean Engineering Branch, Code 3321

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